

Indian Statistical Institute, Bangalore
B. Math II, First Semester, 2021-22
Back Paper Examination, Statistics I
Maximum Score 70

27.01.22

Duration: 2 Hours

Students are allowed to consult the book Statistics by McClave and Sincich.

Values from the normal distribution $qnorm(0.9)=1.28$, $qnorm(.975)=1.96$, $qnorm(0.95)=1.64$
Values from the t distribution $qt(.95,9)=1.833113$, $qt(.95,10)=1.81$, $qt(.975,9)=2.26$, $qt(.975,10)=2.23$,
 $qt(.9,9)=1.38$, $qt(.9,10)=1.37$

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1. (10+2+3) Let $X_1, \dots, X_n$ be a random sample from a $uniform(\theta_1, \theta_2)$ distribution.
 - (a) Find the method of moments estimators of θ_1 and θ_2 .
 - (b) Is this the only MoM estimator, or is it possible to obtain other estimators based on the method of moments?
 - (c) Is the estimator consistent? Justify your answer.
2. (5+5+7) Let $X_1, X_2, \dots, X_n$ be a random sample from the population with distribution function F . Let $F_n(x)$ be the empirical distribution function, that is

$$F_n(x) = \frac{\#\{X_i \leq x\}}{n}$$

Let $c < d$ be two given real numbers. Define $V = F_n(c)$ and $W = F_n(d)$.

- (a) Find the expectation and variance of V .
 - (b) Find the distribution of V
 - (c) Find the covariance between V and W .
3. (5+7+8) Consider the regression model
$$y_i = \beta x_i + \epsilon_i, \quad 1 \leq i \leq n,$$
where ϵ_i are iid with mean zero and variance σ^2 and x_i are fixed.
Consider estimators of β of the form $T\mathbf{a} = \sum_{i=1}^n a_i y_i$, for $\mathbf{a} = (\mathbf{a}_1, \dots, \mathbf{a}_n)^T \in \mathbb{R}^n$.
 - (a) Find the least squares estimator T of β .
 - (b) Find the mean and variance of $T\mathbf{a}$ in terms of a_i 's and the parameters β and σ .
 - (c) Show that estimator T minimizes the variance of $T\mathbf{a}$ over all possible $\mathbf{a} \in \mathbb{R}^n$ such that $T\mathbf{a}$ is unbiased.

4. (4+4+2+4+4) A certain company of battery claims that the average life of a certain type of their battery is 80 weeks. The average life of each of 10 randomly selected batteries is listed below.

84.5, 80.0, 77.3, 81.0, 80.2, 80.1, 78.3, 79.9, 77.8, 75.4

Assume the battery life distribution is normal. Using $\alpha = 0.05$ perform the appropriate test of hypothesis to determine if the average life is smaller than 80 weeks.

- (a) State the null and alternative hypotheses.
- (b) State the test statistic and find its distribution under the null hypothesis.
- (c) Is the distribution in the previous part exact or approximate?
- (d) Find the critical region and compute the value of the test statistic.
- (e) Is the null hypothesis rejected? What is the conclusion regarding the mean battery life?